Closing Tues: HW 12.4, 13.2

Exam 2 is next Thursday!

Covers:

10.1-10.3: Analyzing a function

11.1,2: Deriv. of ln(x) and e^x

12.1,3,4: Integrals, finding C

13.2: Definite Integrals

Entry Task (from HW):

$$AC'(q) = -\frac{4}{q^2} + \frac{1}{4},$$

$$AC(4) = 10$$

Find $AC(q)$, $TC(q)$, and FC.

Total Values and Marginal Values	Total Values and Average Values
$TR(x) = \int MR(x)dx$ $TR'(x) = MR(x)$	$AR(x) = \frac{TR(x)}{x}$ $TR(x) = xAR(x)$
$TC(x) = \int MC(x)dx$ $TC'(x) = MC(x)$	$AC(x) = \frac{TC(x)}{x}$ $TC(x) = xAC(x)$
$P(x) = \int MP(x)dx$ $P'(x) = MP(x)$	

Initial conditions: $TR(0) = 0, TC(0) = FC$

Note: $P(x) = TR(x) - TC(x)$

Section 13.2: Definite integrals and the Fundamental Theorem of Calculus

Recall: An *indefinite integral* is a function (the general antiderivative)

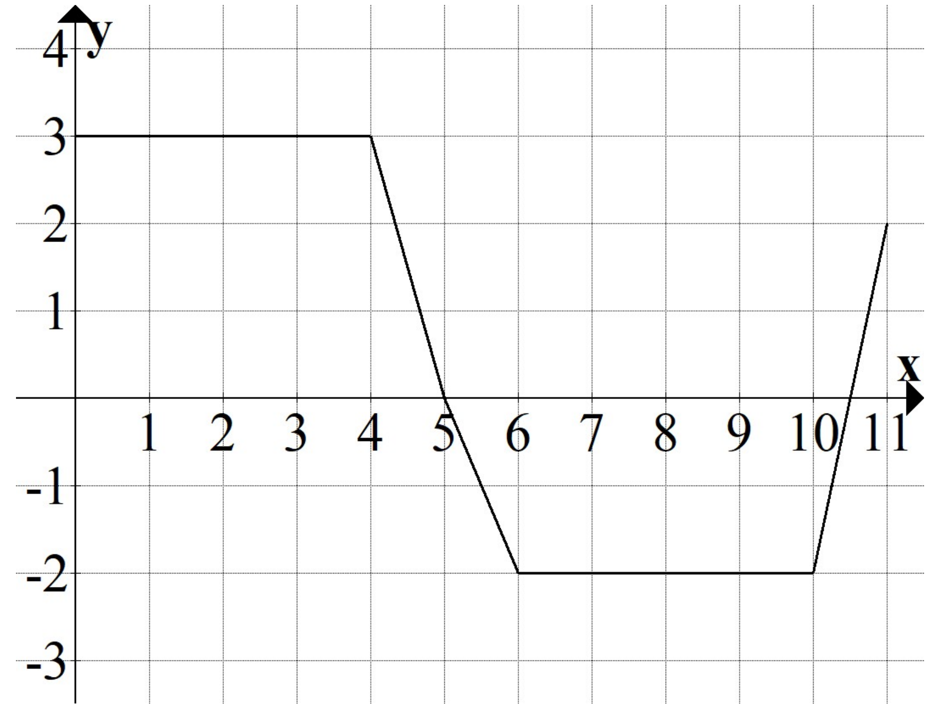
$$\int f(x)dx = F(x) + C$$

New: A *definite integral* is a number that represents *net* area

$$\int_a^b f(x)dx = \text{"}\textit{net}\text{ area between } f(x) \text{ and the } x-\text{axis from } x=a \text{ to } x=b\text{"}$$

<u>*Notes*</u>

Above the x-axis counts as positive area.
Below the x-axis counts as negative area.
"*a*" and "*b*" are called the *bounds,* or *limits*, *of integration*.



$$\int_4^5 f(x)dx =$$

$$\int_5^{10} f(x)dx =$$

$$\int_{10}^{11} f(x)dx =$$

Now consider

$$A(m) = \int_0^m f(x)dx$$

= "*accumulated* net area from *0* to *m*"

Using the same graph, what is

$$A(0) = \int_0^0 f(x)dx =$$

$$A(4) = \int_0^4 f(x)dx =$$

$$A(5) = \int_0^5 f(x)dx =$$

$$A(8) = \int_0^8 f(x)dx =$$

Questions/Observations:

Where is *A(m)* increasing/decreasing?

See any connections for *A(m)* and *f(x)*?

What does $A(5) - A(4)$ represent?

In addition, in the activities you found:

1. "the area under the speed graph"

   equals "the change in distance".

$$\int_a^b s(t)dt = D(b) - D(a)$$

2. "the area under the MR/MC graph"
   equals "the change in TR/TC"

$$\int_a^b MR(x)dx = TR(b) - TR(a)$$

$$\int_a^b MC(x)dx = TC(b) - TC(a)$$

These are examples of a profound fact about anti-derivatives and areas.

The Fundamental Theorem of Calculus

If $F(x)$ is *any* anti-derivative of $f(x)$, then

$$\int_a^b f(t)dt = F(b) - F(a)$$

Example: Find the area under

$$MR(x) = 10 - 2x$$

from $x = 0$ to $x = 4$.

How to compute definite integrals

Step 1: Find *any* antiderivative, $F(x)$.

(usually we pick C = 0, but you use any C value you want and it doesn't change the answer)

Step 2: Evaluate $F(x)$ at $x = b$ and $x = a$.

Step 3: Subtract

We do all this in one line as follows:

$$\int_a^b f(x)dx = \quad F(x)\Big|_a^b = F(b) - F(a)$$

More Examples:

1. $\int_1^2 6x^2 - 2x + 5\, dx$

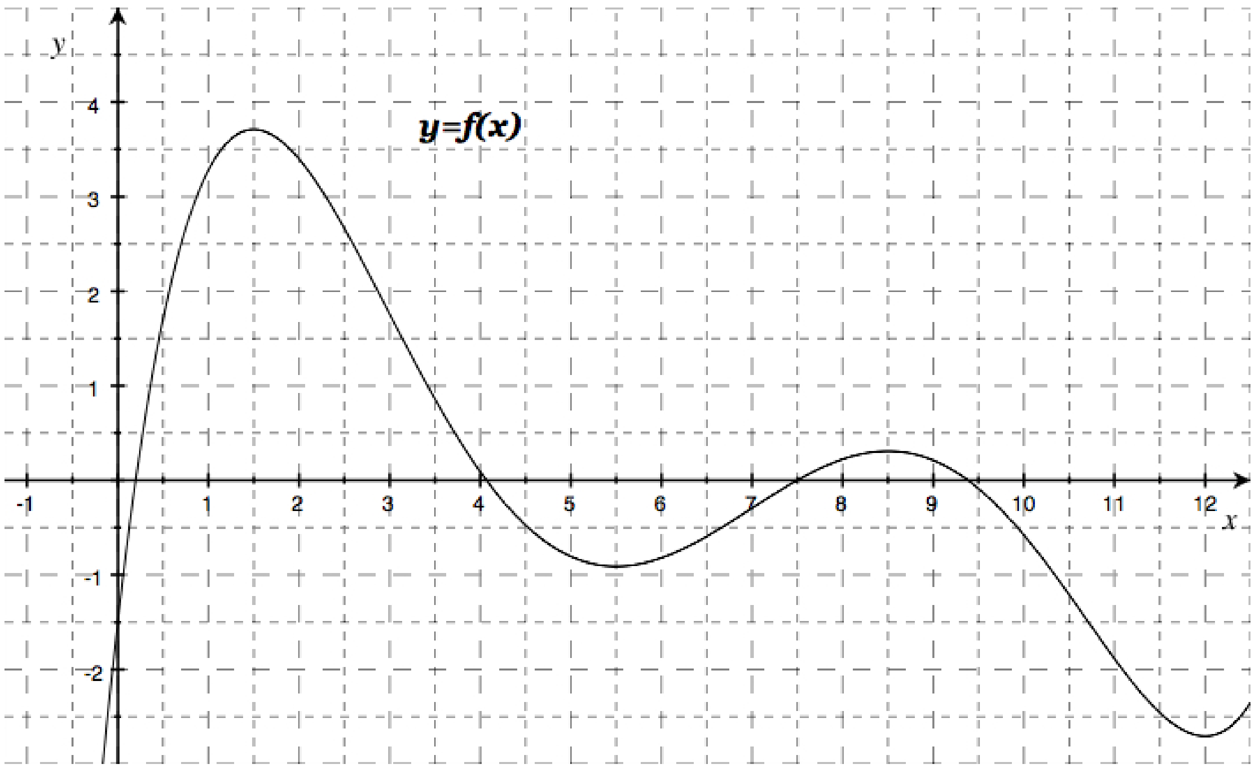
2. $\displaystyle\int_{1}^{5} \frac{3}{4x^2}\, dx$

3. $\displaystyle\int_{0}^{1} e^{x/3}\, dx$

4. $\int_{1}^{4} \sqrt{x}\, dx$

5. $\int_{1}^{e} \frac{5}{x}\, dx$

3. (18 points) Below is the graph of a function $y = f(x)$.



Define the function $A(m)$ by $A(m) = \int_0^m f(x)\,dx$.

NOTE: You do **not** need to show any work for the problems **on this page.**

(a) Name all values of m at which $A(m)$ has a local minimum.

ANSWER: $m =$ ____________________

(b) Give the one-minute interval over which $A(m)$ increases the most.

ANSWER: from __________ to __________

(c) True or False?

circle one

T **F** $A(2.51) > A(2.50)$

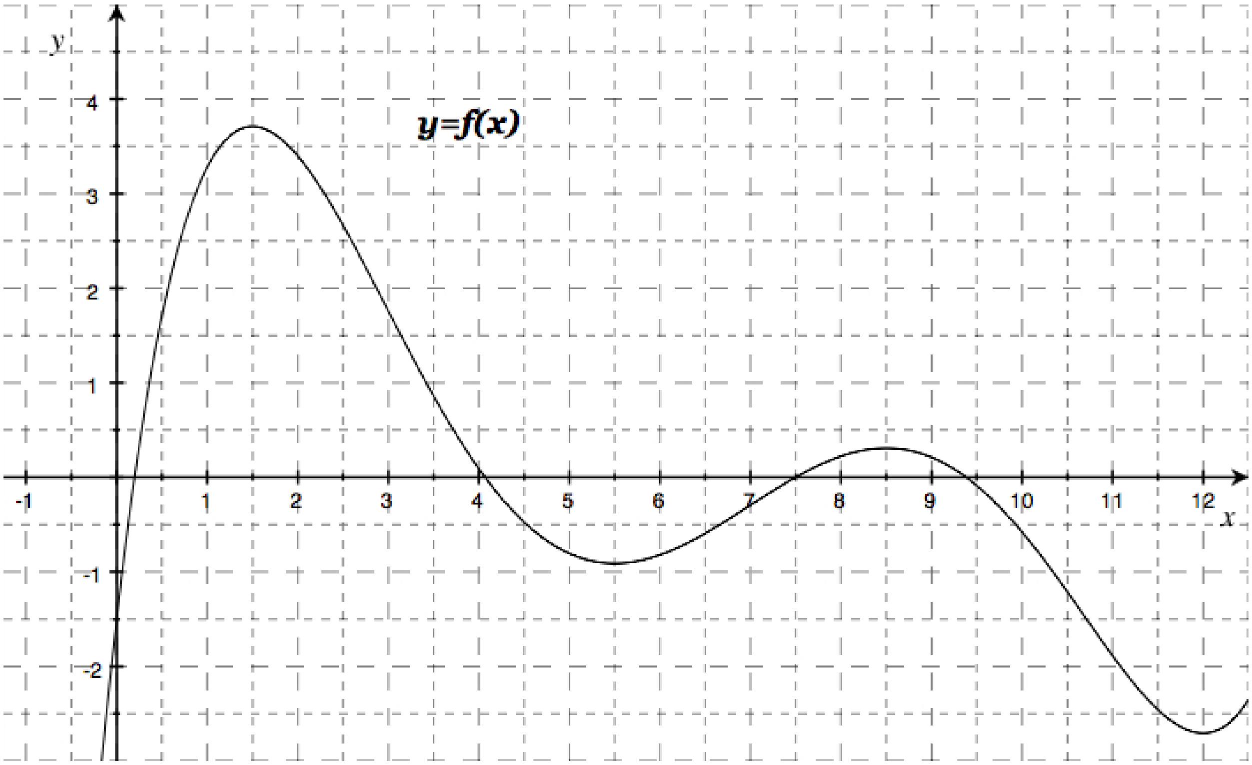
T **F** $f(2.51) > f(2.50)$

T **F** $A(10.01) > A(10.00)$

T **F** $f'(1.00) > f'(1.01)$

(THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.)

Here is the graph of $y = f(x)$ again.



And, again, $A(m) = \int_0^m f(x)\,dx$.

<u>NOTE:</u> The problems on this page **require some justification**: clearly mark points and lines on the graph, shade areas, show calculations of slopes and areas, etc.

(e) Compute $A(1)$.

ANSWER: $A(1) =$ ____________________

(f) Compute $A'(12)$.

ANSWER: $A'(12) =$ ____________________

(g) Compute $A''(5)$.

ANSWER: $A''(5) =$ ____________________

(h) Name a value of x at which $f(x) = f(7)$.

ANSWER: $x =$ ____________________

(i) Compute $A(4) - A(2)$.

ANSWER: $A(4) - A(2) =$ ____________________